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On the convective instability of a horizontal binary mixture layer with Soret effect under transversal high frequency vibration

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Abstract

The stability of the mechanical equilibrium of a plane horizontal binary mixture layer with Soret effect in the presence of high frequency transversal vibration is studied.

The asymptotic analysis for long-wave disturbances and the numerical solution of the spectral amplitude problem for cellular disturbance has shown that independently of properties of the binary mixture the effect of transversal vibration is always stabilizing. The critical values of Rayleigh number and the characteristics of critical disturbances are determined.

Nomenclature

b displacement amplitude
C deviation of concentration from reference value $\bar{C}$
D coefficient of diffusion
$f(z)$ amplitude of F-disturbances
F stream function for oscillatory part of the velocity
g acceleration of gravity
h thickness of the layer
$\mathbf{j}$ flux of lightest component
k wave number
Le Lewis number
$\mathbf{n}$ unit vector along axis of vibration
p pressure
Pr Prandtl number
Ra Rayleigh number
Ra_v non-dimensional vibrational parameter
Sc Schmidt number
$\mathbf{v}$ (v_x, v_y, v_z) mean velocity
$\mathbf{w}$ (w_x, w_y, w_z) oscillatory velocity
(x, y, z) Cartesian coordinates.

Greek symbols

α thermodiffusional ratio
β_1 coefficient of thermal expansion
β_2 concentrational coefficient of density
γ unit vector along z-axis
ε non-dimensional Soret parameter
θ amplitude of T-disturbances
Θ difference of temperature
λ decrement
ν kinematic viscosity
ξ amplitude of C-disturbances
ρ density
τ period of vibration
$\varphi(z)$ amplitude of ψ-disturbances
χ coefficient of heat diffusivity
ψ stream function for mean part of the velocity
Ω angular frequency of vibration.

1. Introduction

It is well known that vibration of a cavity filled with fluid has a strong effect on the convective flows in the presence of non homogeneous temperature distribution.

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In some cases the vibration can provoke a mean flow with the structure and intensity depending on the direction and characteristics of the vibration. Thus, in the general case it is possible to distinguish two mechanisms of thermal convection excitation—thermogravitational and thermovibrational. The problem of thermovibrational convective stability of mechanical equilibrium in the state of weightlessness has been investigated [1–4].

The problem of vibrational convective instability for the case of a binary mixture with Soret effect has been investigated recently [5]. The effect of longitudinal high frequency vibration on a plane horizontal layer of the mixture, with rigid and isothermal boundaries, was studied. It was shown that convective instability is caused by both mechanisms of excitation—thermogravitational and thermovibrational, which are superimposed.

In the present paper we consider the case where the axis of vibration is vertical, i.e. transversal with respect to the layer. Physically the situation is quite different from the one considered in [5]. For a one component medium it has been proven [2] that the equilibrium is absolutely stable if the axis of vibration is parallel to the temperature gradient. Thus one expects that for a binary mixture the mechanical equilibrium will be stable if the axis of vibration and the density gradient are mutually parallel. The analysis performed confirms this expectation.

In Section 2 the problem is described and the main equations are written down. In Section 3 the state of mechanical equilibrium is considered and the problem of its linear convective stability is formulated. In Section 4 the asymptotic analysis is developed for the limiting case of long-wave normal disturbances. The wave number of the normal mode is used as a small parameter for regular perturbation theory. In Section 5 the numerical results for arbitrary wave numbers (cellular modes) are presented and discussed.

2. The problem description and the basic equations

We consider an infinite plane horizontal binary mixture layer with Soret effect. The horizontal boundaries of the layer are assumed to be rigid, isothermal and impermeable to the mixture components. The Cartesian coordinate system is chosen with the origin on the lower plate $z = 0$ and with the z-axis directed vertically upward. The temperature of the lower plane $z = 0$ is maintained constant and equal to Θ, the temperature of the upper plane $z = h$ is chosen as a reference point 0, so both cases $\Theta > 0$ and $\Theta < 0$ will be considered corresponding to heating from below or from above. There is a static gravity field with acceleration $\mathbf{g}\ (0, 0, -g)$ and also the high frequency vibration with an axis which is parallel to the z-axis.

There is no external difference of concentration and the inhomogeneity of concentration which appears is only caused by the temperature gradient and the Soret effect. We will see later that in the situation described, the specific vibrational mechanism of instability excitation is not operative, so the only reason for instability is the potentially unstable density stratification in a static gravitational field. Thus the instability mechanism expected is of the Rayleigh–Benard nature. To describe the thermogravitational and concentration gravitational convection we therefore assume that the standard Boussinesq approximations are valid. Thus the equation of the state is of the form

$$\rho = \bar{\rho}(1 - \beta_1 T - \beta_2 C) \tag{2.1}$$

where ρ and $\bar{\rho}$ are the fluid density and its standard value, respectively; T is the temperature; C is the concentration of the lightest component (T and C are only slightly different from the 'constant' values T and C); $\beta_1 > 0$ the thermal expansion coefficient; and $\beta_2 > 0$ the concentrational density coefficient.

The equation of motion in the non inertial proper coordinate system, connected with the vibrating layer must take into account the vibrational acceleration, i.e. the gravitational acceleration $\mathbf{g}$, must be replaced by

$$\mathbf{g} \to \mathbf{g} + b\Omega^2 \cos \Omega t \mathbf{n} \tag{2.2}$$

where b is the displacement amplitude, Ω is the angular frequency and $\mathbf{n}$ is the unit vector in the direction of the vibration axis.

Then we have the equation of motion in the form

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p + \nu \Delta \mathbf{v} + g(\beta_1 T + \beta_2 C)\gamma + (\beta_1 T + \beta_2 C) b\Omega^2 \cos \Omega t \mathbf{n} \tag{2.3}$$

where $\mathbf{v}$ is the velocity, ν is the kinematic viscosity and $\boldsymbol{\gamma}$ is the unit vector along the z-axis. The expression for the flux of lightest component is used to elaborate the equation for concentration taking into account the Soret effect:

$$\mathbf{j} = -\bar{\rho} D(\nabla C + \alpha \nabla T) \tag{2.4}$$

where D is the diffusion coefficient and α is the thermodiffusional ratio ($\alpha > 0$ and $\alpha < 0$ correspond to the anomalous and normal Soret effect, respectively). We suppose that both coefficients are constant, so we have the equation for concentration

$$\frac{\partial C}{\partial t} + \mathbf{v}\nabla C = D(\Delta C + \alpha \Delta T). \tag{2.5}$$

The heat transport and continuity equations are written as in the standard Boussinesq model

$$\frac{\partial T}{\partial t} + \mathbf{v}\nabla T = \chi \Delta T \tag{2.6}$$

$$\operatorname{div} \mathbf{v} = 0 \tag{2.7}$$

where χ is the heat diffusivity coefficient.

In the asymptotic case of high frequency and small amplitude the averaging method can be applied to obtain the closed system of equations for mean fields—velocity, temperature, pressure and concentration. The details of the averaging procedure are rather standard (see [6]) and are omitted here.

The system of equations for mean fields can be written in the form:

$$\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p+\nu\Delta\mathbf{v}+g(\beta_1 T+\beta_2 C)\gamma + \frac{1}{2}b^2\Omega^2(\mathbf{w}\nabla)[(\beta_1 T+\beta_2 C)\mathbf{n}-\mathbf{w}] \quad (2.8)$$

$$\frac{\partial T}{\partial t}+\mathbf{v}\nabla T = \chi\Delta T \quad (2.9)$$

$$\frac{\partial C}{\partial t}+\mathbf{v}\nabla C = D(\Delta C+\alpha\Delta T) \quad (2.10)$$

$$\operatorname{div}\mathbf{v} = 0 \quad (2.11)$$

$$\operatorname{div}\mathbf{w} = 0, \operatorname{curl}\mathbf{w} = \nabla(\beta_1 T+\beta_2 C)\times\mathbf{n}, \quad (2.12)$$

where $\mathbf{v}$, p, T and C are the mean parts of corresponding fields. All the mean fields are the function slowly varying with time (the characteristic time is much longer than the vibration period τ), $\mathbf{w}$ is an additional variable which varies slowly with time. It is the solenoidal part of the vector $(\beta_1 T+\beta_2 C)\mathbf{n}$. It is also the slowly varying time dependent amplitude of the oscillatory (quick) part of the velocity filed $\mathbf{v}$, as defined by

$$\mathbf{v}' = b\Omega\sin\Omega t\mathbf{w}. \quad (2.13)$$

Let us now define the equations system for the mean field in non dimensional form, using the following units: h for distance, h^2/ν for time, χ/h for velocity, Θ for temperature, $\beta_1\Theta/\beta_2$ for concentration, $\beta_1\Theta$ for $\mathbf{w}$ and $\rho\nu\chi/h^2$ for pressure. Thus the governing system is:

$$\frac{\partial \mathbf{v}}{\partial t}+\frac{1}{Pr}(\mathbf{v}\nabla)\mathbf{v} = -\nabla p+\Delta\mathbf{v}+Ra(T+C)\gamma + Ra_v(\mathbf{w}\nabla)[(T+C)\mathbf{n}-\mathbf{w}], \quad (2.14)$$

$$Pr\frac{\partial T}{\partial t}+\mathbf{v}\nabla T = \Delta T \quad (2.15)$$

$$Sc\frac{\partial C}{\partial t}+\frac{Sc}{Pr}\mathbf{v}\nabla C = \Delta(C-\varepsilon T) \quad (2.16)$$

$$\operatorname{div}\mathbf{v} = 0 \quad (2.17)$$

$$\operatorname{div}\mathbf{w} = 0, \operatorname{curl}\mathbf{w} = \nabla(T+C)\times\mathbf{n}. \quad (2.18)$$

The problem includes the following set of non-dimensional parameters: the Rayleigh number Ra, which is positive if the system is heated from below and negative if it is heated from above; the vibrational parameter Ra_v (the vibrational analog of the Rayleigh number) which is always positive, the Prandtl number Pr and the Schmidt number Sc, the non dimensional parameter of the Soret effect ϵ, which is either positive (normal effect) or negative (anomalous effect). The parameters are determined as:

$$Ra = \frac{g\beta_1\Theta h^3}{\nu\chi}, Ra_v = \frac{(b\Omega\Theta h\beta_1)^2}{2\nu\chi}$$

$$Pr = \frac{\nu}{\chi}, Sc = \frac{\nu}{D}, \epsilon = -\frac{\alpha\beta_2}{\beta_1}. \quad (2.19)$$

Let us formulate the boundary conditions for the horizontal binary mixture layer with rigid, isothermal and impermeable boundaries

$$\text{at } z = 0 \quad \text{and} \quad z = 1: \mathbf{v} = 0, w_z = 0, \frac{\partial C}{\partial z}-\epsilon\frac{\partial T}{\partial z} = 0$$

$$\text{at } z = 0: T = 1,$$

$$\text{at } z = 1: T = 0. \quad (2.20)$$

The physical assumptions connected with the performances of the averaging are:

(i) the frequency must be high (but not acoustic), thus the period of vibration must be small with respect to all the characteristic hydrodynamic times

$$\tau \ll \min\left(\frac{h^2}{\nu}, \frac{h^2}{\chi}, \frac{h^2}{D}\right) \quad (2.21)$$

(ii) the displacement amplitude must be small with respect to the ratio between the thickness of the layer and the Boussinesq parameter

$$b \ll \frac{h}{\beta_1\Theta} \quad (2.22)$$

(iii) the accelerations must be related by

$$\frac{g\beta_1\Theta}{\Omega^2 h} \ll 1. \quad (2.23)$$

3. Mechanical equilibrium and stability problem formulation

An important question is whether the state of mechanical quasiequilibrium (i.e. the state at which the mean velocity is zero, but the pulsational component is not in general) exists or not in our situation.

To find the quasiequilibrium conditions it is necessary to set up $\mathbf{v} = 0$, $\partial/\partial t = 0$, $P = P_0$, $T = T_0$, $C = C_0$ and $\mathbf{w} = \mathbf{w}_0$, where P_0, T_0, C_0 and $\mathbf{w}_0$ are the distributions in the state of mechanical equilibrium. The general system leads to the following necessary conditions for quasi-equilibrium fields:

$$\nabla(T_0+C_0)\times[Ra\gamma-Ra_v\nabla(\mathbf{w}_0\mathbf{n})] = 0\Delta T_0 = 0, \quad \Delta C_0 = 0\operatorname{div}\mathbf{w}_0 = 0, \operatorname{curl}\mathbf{w}_0 = \nabla(T_0+C_0)\times\mathbf{n} \quad (3.1)$$

with appropriate boundary conditions (2.20).

It is easy to see that when the axis of vibration is vertical ($\mathbf{n} = \gamma$), the state of quasiequilibrium exists and its structure is very simple:

$$T_0 = 1-z, \frac{\mathrm{d}C_0}{\mathrm{d}z} = -\epsilon, \quad \mathbf{w}_0 = 0. \quad (3.2)$$

The profiles of temperature and concentration are linear and $\mathbf{w} = 0$. That means that we have to deal now with the situation of complete (not 'quasi') mechanical equilibrium, i.e. not only when the mean but also the oscillatory components of the velocity are equal to zero.

To study the stability of mechanical equilibrium let us consider small two dimensional disturbances of all the equilibrium fields:

$T_0 + T', C_0 + C', p_0 + p', \mathbf{w}_0 + \mathbf{w}', \mathbf{v}.$

The system of equations for disturbances is obtained from the general system [Equations (2.14)–(2.18)] by linearization near the equilibrium solution (3.2). (Further, the superscript primes will be omitted.)

Let us consider 2-D-disturbances and introduce the stream functions for the mean and oscillatory components of the velocity:

$$v_x = \frac{\partial \psi}{\partial z}, v_z = -\frac{\partial \psi}{\partial x}, w_x = \frac{\partial F}{\partial z}, w_z = -\frac{\partial F}{\partial x} \tag{3.3}$$

and formulate the system of equations for disturbances in terms of ψ, F, T and C:

$$\frac{\partial}{\partial t}\Delta\psi = \Delta^2\psi - Ra\left(\frac{\partial T}{\partial x} + \frac{\partial C}{\partial x}\right) - (1+\epsilon)Ra_v\frac{\partial^2 F}{\partial x^2}$$

$$Pr\frac{\partial T}{\partial t} + \frac{\partial \psi}{\partial x} = \Delta T Sc\frac{\partial C}{\partial t} + \epsilon\frac{Sc}{Pr}\frac{\partial \psi}{\partial x} = \Delta(C - \epsilon T)$$

$$\Delta F = -\frac{\partial}{\partial x}(T + C) \tag{3.4}$$

here Δ is a 2-D Laplace-operator in the plane (x, z). We will consider the disturbances in the form of normal modes:

$$(\psi, T, C, F) = (\varphi(z), \theta(z), \xi(z), f(z))\exp(-\lambda t - ikx). \tag{3.5}$$

Here λ is the decrement, k is the wave number, $\varphi(z)$, $\theta(z)$, $\xi(z)$ and $f(z)$ are the amplitudes. After substitution of (3.5) into (3.4) we obtain the system of amplitude equations:

$$-\lambda\mathscr{D}\varphi = \mathscr{D}^2\varphi + ikRa(\theta + \xi) + (1+\epsilon)k^2 Ra_v f$$

$$-\lambda Pr\theta - ik\varphi = \mathscr{D}\theta$$

$$-\lambda Sc\xi - ik\epsilon\frac{Sc}{Pr}\varphi = \mathscr{D}\xi - \epsilon\mathscr{D}\theta$$

$$\mathscr{D}f = ik(\theta + \xi), \mathscr{D} = \frac{d^2}{dz^2} - k^2 \tag{3.6}$$

with the boundary conditions:

at $z = 0$ and $z = 1$:

$$\varphi = 0, \varphi' = 0, f = 0, \theta = 0, \xi' - \epsilon\theta' = 0. \tag{3.7}$$

The prime denotes differentiation with respect to the transversal coordinate z. Thus we have a spectral amplitude problem with decrement λ as eigenvalue, depending on all the parameters:

$$\lambda = \lambda(Ra, Ra_v, \epsilon, Pr, Sc, k). \tag{3.8}$$

In general, λ is complex, $\lambda = \lambda_r + i\lambda_i$. $\lambda_i = 0$ corresponds to the case of monotonous behavior of the disturbance and the boundary of stability can be determined from the condition $\lambda = 0$. The case $\lambda_i \neq 0$ corresponds to the oscillatory form of disturbance, and the condition $\lambda_r = 0$ determines the boundary of oscillatory instability, whereas the imaginary part λ_i gives the frequency of neutral critical disturbance.

In the limiting case of steady instability ($\partial/\partial t = 0$) the problem (3.4) can be re-formulated. As a result only three parameters specifies the instability boundary. To obtain the system of equations with a reduced number of parameters a new variable is introduced: $H = C + \alpha T$. Equation (2.10) then takes the form

$$\frac{\partial H}{\partial t} + \mathbf{v}\nabla H = D\Delta\left(H + \frac{\alpha\chi}{D}T\right).$$

The other equations of the system (2.8)–(2.12) are also transformed. In non-dimensional notation, it is worthwhile to choose alternate scales for the concentration and $\mathbf{w}$, let us take $\beta_1\theta(1+\varepsilon)/\beta_2$ for H and $\beta_1\theta(1+\varepsilon)$ for $\mathbf{w}$. Skipping the intermediate steps, the following system of equations under steady conditions can be written instead of (3.4):

$$\Delta^2\psi - \widetilde{Ra}\left(\frac{\partial H}{\partial x} + \frac{\partial T}{\partial x}\right) - \widetilde{Ra}_v\frac{\partial^2 F}{\partial x^2} = 0$$

$$\Delta T - \frac{\partial \psi}{\partial x} = 0$$

$$\Delta(H - \varphi T) = 0$$

$$\delta F + \frac{\partial}{\partial x}(H + T) = 0.$$

Here $\delta\psi = \psi^{\mathrm{IV}} - 2k^2\psi'' + k^4\psi$.

Thus a stability boundary is determined then by three parameters:

$$\widetilde{Ra} = Ra(1+\varepsilon), \quad \widetilde{Ra}_v = Ra_v(1+\varepsilon)^2 \quad \varphi = \frac{Sc}{Pr}\frac{\varepsilon}{1+\varepsilon}.$$

The situation is similar to that investigated by Gutkowicz-Krusin et al. [7]. Nevertheless we find it expedient to keep the five parameters when discussing the results. It allows an easier and a better physical understanding and facilitate the comparison with the case of the longitudinal orientation of the axis of vibration [5].

4. The limiting case of long-wave disturbances

In the general case the solution of the eigenvalue problem (3.6), (3.7) must be found numerically. But the condition of impermeability enables us to expect that for some ranges of parameter values, long-wave disturbances (with the wave number $k = 0$) are responsible for instability excitation. To study the behaviour of long-wave disturbances one may develop the regular perturbation method with the wave number k as a small parameter.

So, let us try to construct the asymptotic solution of the spectral amplitude problem (3.6), (3.7) in the form of power expansions of all the amplitudes and the eigenvalue:

$$\varphi = \varphi_0 + k\varphi_1 + k^2\varphi_2 + \cdots$$
$$\theta = \theta_0 + k\theta_1 + k^2\theta_2 + \cdots$$
$$\xi = \xi_0 + k\xi_1 + k^2\xi_2 + \cdots$$
$$f = f_0 + kf_1 + k^2 f_2 + \cdots$$
$$\lambda = \lambda_0 + k\lambda_1 + k^2\lambda_2 + \cdots \quad (4.1)$$

In a standard way we obtain the systems of equations of successive approximations [the boundary conditions in each approximation coincide with (3.7)].

The system of equations in the zeroth order is:

$$-\lambda_0\varphi_0'' = \varphi_0^{\mathrm{IV}}$$
$$-\lambda_0\theta_0 Pr = \theta_0''$$
$$-\lambda_0\xi_0 Sc = \xi_0'' - \in\theta_0''$$
$$f_0'' = 0. \quad (4.2)$$

The inspection of the zero-order spectral problem shows that only one non trivial level exists:

$$\lambda_0 = 0, \varphi_0 = 0, \theta_0 = 0, f_0 = 0, \xi_0 = \text{const} \quad (4.3)$$

where const can be determined from the condition of normalization. This level is neutral and it is of concentrational type.

For the first order we have the non homogeneous system:

$$\varphi_1^{\mathrm{IV}} = -iRa\xi_0$$
$$\theta_1'' = 0$$
$$\xi_1'' = -\lambda_1 Sc\xi_0$$
$$f_1'' = i\xi_0. \quad (4.4)$$

The condition of solvability gives $\lambda_1 = 0$. The solution is:

$$\lambda_1 = 0, \theta_1 = 0, \xi_1 = \text{const}$$
$$f_1 = \frac{i\xi_0}{2}z(z-1), \varphi_1 = -\frac{iRa\xi_0}{24}z^2(z-1)^2. \quad (4.5)$$

The amplitude system of the second order is:

$$\varphi_2^{\mathrm{IV}} = -iRa\xi_1$$
$$\theta_2'' = -i\varphi_1$$
$$\xi_2'' = \xi_0 - i\in\varphi_1\left(1+\frac{Sc}{Pr}\right) - \lambda_2 Sc\xi_0$$
$$f_2'' = i\xi_1. \quad (4.6)$$

Only the solvability condition for this non-homogeneous system is required. This condition can be obtained by integration of the equation for ξ_2 with respect to z from $z = 0$ up to $z = 1$.

This allows us to determine λ_2, the first non-vanishing approximation to the decrement

$$\lambda_2 = \frac{1}{Sc}\left(1 - \in\frac{Sc}{Pr}\frac{Ra}{720}\right). \quad (4.7)$$

So we see that the decrement is real and the disturbances are monotonous in the long-wave limiting case. The stability boundary is determined from the condition $\lambda_2 = 0$ and the critical value of Ra for the long wave mode is:

$$Ra = \frac{720Le}{\in}, \quad (4.8)$$

where $Le = Pr/Sc$, is the Lewis number.

Thus, the critical value of the Rayleigh number for the long wave mode does not depend on the vibrational parameter Ra_v, but it depends mostly on the Soret parameter $\in$.

Long wave excitation is possible when the system is heated from below in the case of the normal Soret effect and also when the system is heated from above—in the case of the anomalous effect.

To judge whether the long wave mode is more dangerous or not it is necessary to determine the stability boundary for cellular modes (with finite values of the wave number k). This requires the solution of the complete spectral problem (3.6), (3.7).

Some results of the numerical solution of this problem are presented in the next section.

5. Numerical results

The numerical solution of the complete spectral eigenvalue problem has been obtained by straight forward step-by-step numerical integration of the system of amplitude equations by the Runge–Kutta–Merson method in combination with shooting procedure. Some numerically determined instability boundary characteristics are presented: the critical values of the Rayleigh number Ra_m (obtained as a result of minimization of critical Ra with respect to the wave number k), the critical wave number k_m (corresponding to the minimum of the neutral curve $R(k)$) and the critical frequency value of the most dangerous frequency λ_{im} (in the case of oscillatory instability). In Figs 1–4 the numerical results are presented in the form of stability curves in the plane ε–Ra_m (i.e. the minimal critical Rayleigh number Ra, a function of the non-dimensional Soret parameter $\in$) for a values of the vibrational parameter Ra_v and for a few combination of Pr and Sc corresponding to typical gaseous and liquid binary mixtures. In the upper parts of the figures the data concerning k_m and λ_{im} are also presented. In all the figures the solid lines correspond to the stability boundary with

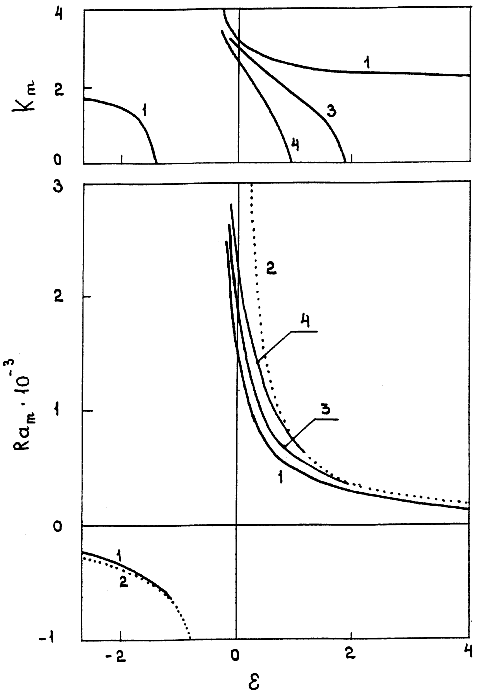

Fig. 1. The instability parameters for $Pr = 1$ and $Sc = 1$. The numeration of the curves: 1—$Ra_v = 0$, 2—$k_m = 0$, 3—$Ra_v = 200$, 4—$Ra_v = 1000$.

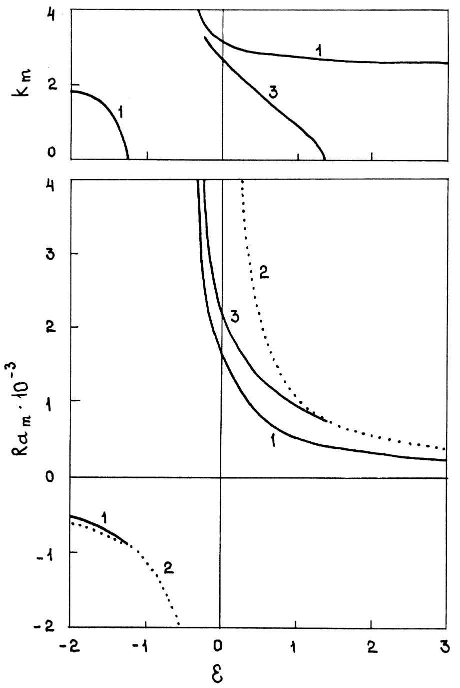

Fig. 2. The instability for $Pr = 0.75$ and $Sc = 0.5$. The numeration of the curves: $1-Ra_v = 0$, $2-k_m = 0$, $3-Ra_v = 1000$.

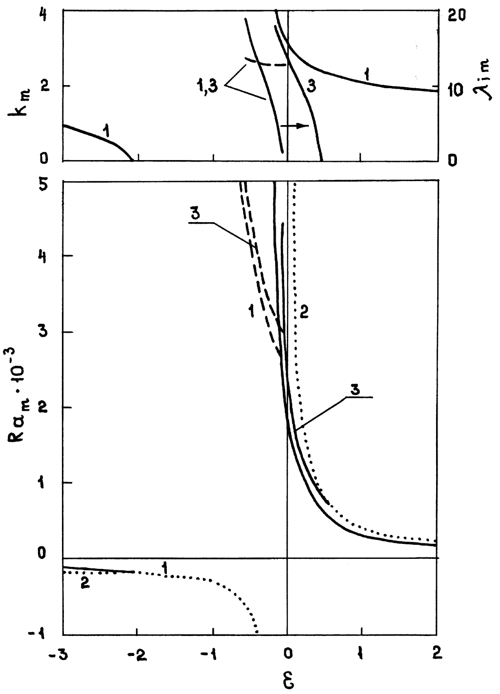

Fig. 3. The instability parameters for $Pr = 0.75$ and $Sc = 1.5$. The numeration of the curves: $1-Ra_v = 0$, $2-k_m = 0$, $3-Ra_v = 1000$.

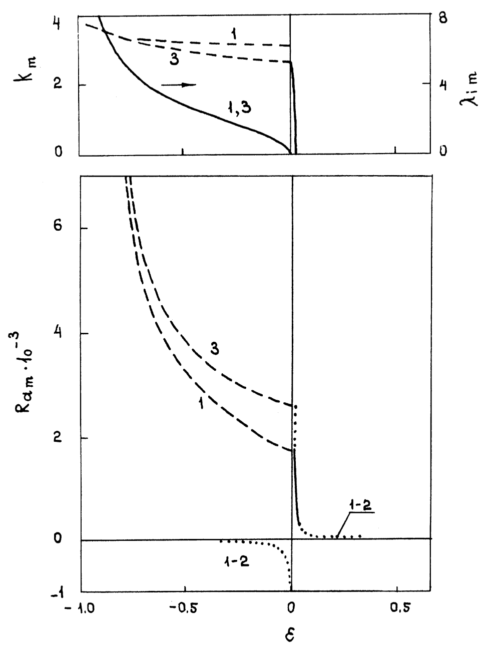

Fig. 4. The instability parameters for $Pr = 6.7$ and $Sc = 677$. The numeration of the curves: $1-Ra_v = 0$, $2-k_m = 0$, $3-Ra_v = 2000$.

respect to the monotonous cellular modes, the dashed lines—to oscillatory cellular modes, and the dotted lines—to long wave monotonous modes. The dotted lines presented in Figs 1–4 are determined numerically but in the parameter ranges where the long wave mode is the most dangerous, the critical values of the Rayleigh number coincide with those given by the asymptotic formula (4.8).

First consider the model case of a gaseous binary mixture with the parameter values $Pr = Sc = Le = 1$ (Fig. 1). Oscillatory instability is not possible in this case. Line 1 corresponds to $Ra_v = 0$, where vibration is absent (this problem is not new, see [8–10] and references therein). We see the destabilization due to the Soret effect in the region $\epsilon > 0$ and stabilization at $\epsilon < 0$ when heated from below. There is also the stability line in the region $\epsilon < 0$ when heated from above.

When a vibration is switched on ($Ra_v = 200$—line 3; $Ra_v = 1000$—line 4), the critical values of Ra_m increase indicating vibrational stabilization. All the lines corresponding to $Ra_v \neq 0$ are disposed between two characteristic lines: 1 ($Ra_v = 0$) and 2 ($k_m = 0$). So, the boundary of long wave instability which is described by quation (4.8) is the upper boundary of the stability curves and this condition is valid in other cases, presented in Figs 2–4. Therefore, independently of the parameters, we may say that $Ra > 720\ Le/\epsilon$ is a sufficient criterion for instability under heating from below. When ϵ is small the instability under vibration is of cellular character. For example, at $Ra_v = 200$ the instability is cellular if $\epsilon < 1.9$ but if $\epsilon > 1.9$ transition to the long wave form of instability occurs. For $Ra_v = 1000$ the critical value of ϵ is 0.93.

For $Pr = 0.75$ and $Sc = 0.5$ ($Le = 1.5$) (Fig. 2) the situation is qualitatively the same. As above, only qualitative shiftings are observed. The new element is that oscillatory amplifying modes are now possible but the most dangerous mode is still the monotonous one.

For $Pr = 0.75$ and $Sc = 1.5$ (Fig. 3), i.e. $Le = 0.5 < 1$ oscillatory instability appears as the most dangerous mode for $\epsilon < 0$.

Finally consider the case corresponding to the typical liquid solution, for example, salt-water, namely, $Pr = 6.7$ and $Sc = 677$ (Fig. 4). Note the sharp destabilization of the monotonous mode in the region of $\epsilon > 0$ and the sharp stabilization at $\epsilon < 0$.

When the system is heated from below, in practically the entire region $\epsilon < 0$ the oscillatory modes are responsible for the appearance of instability. In contrast, when the system is heated from above and $\epsilon < 0$, the instability threshold is connected with long wave disturbances.

6. Conclusions

For a binary mixture in an infinite horizontal layer in the presence of vertical high frequency vibration, contrary to the case of longitudinal h.f. vibration, the specific vibrational mechanism of instability by excitation is not operative. The effect of vibration is purely stabilizing: at arbitrary values of binary mixture parameters the critical Rayleigh number increases monotonously with increasing vibrational parameter.

The critical values of Rayleigh number and the characteristics of the critical disturbances are determined analytically (in the case of long wave modes) and numerically (in the case of cellular modes).

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